Preprint Version for 2009 IEEE Conference on Decision and Control

Towards Constant Velocity Navigation and Collision Avoidance for Autonomous Nonholonomic Aircraft-like Vehicles

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Abstract— **This paper presents a methodology for the decentralised control of multiple 3-dimensional nonholonomic agents. The proposed control scheme is based on Navigation Functions and offers improvements compared to previous work of the authors in this field. The problem formulation is chosen to resemble Air-Traffic Management, where the safety guarantees provided by Navigation Functions based control strategies are very important. The linear velocity of each agent is maintained constant and equal to a desired value in most cases, while the azimuth and elevation control laws are engineered to reduce the required control effort. These qualitative improvements do not affect the collision avoidance characteristics of the control strategy. The performance and efficiency of our approach is supported by computer simulations presented in the paper.**

I. INTRODUCTION

Navigation of multiple nonholonomic vehicles is gaining a lot of interest from a control perspective, since it poses a practical, yet challenging problem. Uninhabitated (Ground, Underwater, Aerial) vehicles (UGVs, UAVs, AUVs) and Air Traffic Management (ATM) are some of the most important applications involving nonholonomic agents. Automated ATM, in particular, poses additional difficulties, as the available control inputs are severely limited due to the aircraft's maneuvering capabilities, while effective collision avoidance is most important for flight safety.

The unicycle is usually used to model nonholonomic vehicles, and a variety of methods have been proposed for the control of unicycle-like agents with limited control capacity: Carbone et. al. [1] employ the collision cone concept using one control input (either turn, descent/climb or change the speed), but have not provided any formal guarantee for collision avoidance. Lalish et. al. [2] have also used the collision cone concept in a decentralised algorithm which considers actuation limits, but can not guarantee safe collision avoidance from any initial conditions. Oikonomopoulos et. al. [3] have addressed the problem in a discrete and centralised manner, where actuation limits are satisfied but with an associated high computational cost.

Potential fields, and Navigation Functions [4] in particular have been used for decentralised navigation of planar nonholonmic agents [5], while an extension to 3-dimensional space has been recently proposed [6]. The main advantage of Navigation Functions based methods is the solid formal proof they can provide to support their performance. Constraint satisfaction is very important for certain applications, such as Air Traffic Control. However, integration of input constraints has so far been only limited. In [7] a variable gain approach is presented for holonomic systems, while a combination of Navigation Functions and Model Predictive Control (MPC) for constraint satisfaction has been attempted in [8]. Optimisation methods have also been used for aircraft collision avoidance, in non-cooperative (worst case) schemes [9], [10], as well as decentralised, cooperative approaches [11], [12]. Although optimisation methods are appealing for handling constraints, they suffer from computational issues along with combinatorial increase in complexity. As a result they are unsuitable for real-time implementation in safety critical applications, like aircraft collision avoidance, but can offer useful results for offline maneuver calculations.

In this paper we present a control scheme utilizing Navigation Functions for the decentralised control of multiple nonholonomic agents with velocity constraints. We formulate the problem in a way pertinent to Air Traffic Management (ATM), where each aircraft can monitor the position, orientation and velocity of neighboring aircraft through surveillance. Towards decentralization and communication minimization, we assume for each agent (aircraft) no knowledge of the destinations other than its own. The fact that the method is fully 3D means that each aircraft can use vertical as well as horizontal maneuvering to exploit the available airspace and stay away from conflicts. As the decentralization of Air Traffic Control is thought to be a solution to the increasing air traffic load, the control scheme that follows can be useful in the design of future ATM systems. Another application where such an algorithm may be considered is the case of multiple Autonomous Underwater Vehicles (AUVs) operating in the same area.

Our approach employs Dipolar Navigation Functions [13] in a control scheme driving the agents away from each other and towards their destinations. This new control strategy aims at producing more natural trajectories compared to previous work based on Navigation Functions [6]. This is achieved by maintaining the absolute linear velocity of each agent equal to a desired value (independent for each agent) and avoiding unnecessary turns when possible. The desired absolute velocity used can be constant or regulated independently. The proposed control scheme maintains the desired absolute velocity when possible, and while the agent is sufficiently away from its destination. Whenever this desired velocity could compromise the system's convergence (as checked by a simple inequality), a higher absolute velocity is applied. Finally near destination the velocity is reduced for the final approach. An important result of the above control strategy

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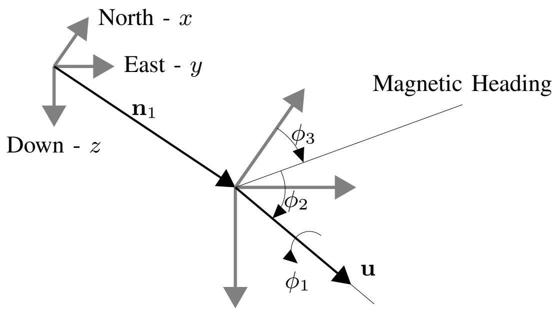

Fig. 1: Earth-Fixed Coordinates

is that the desired velocity acts as a lower bound for the absolute velocity, eliminating infeasibly low velocities, a must for Air Traffic Control. Moreover, under some mild requirements regarding the initial conditions, speed reversals can be eliminated and a specific direction (forward or backward) can be enforced for each agent. Regarding the angular velocities, no actuation is used when not necessary, thus reducing the total control effort. These qualitative improvements achieved by our control strategy do not come at the cost of performance or safety, as the convergence and collision avoidance characteristics are formally verified.

The rest of the paper is organised as follows: section II describes the nonholonomic model used for the agents and the problem treated, followed by an brief introduction to Dipolar Navigation Functions in section III. In section IV, the proposed control scheme is presented, while section V includes computer simulation supporting the derived results and demonstrate the efficiency of our approach. The conclusions of this paper are summarised in section VI.

II. SYSTEM AND PROBLEM DEFINITION

Each agent i is described by a 3-dimensional kinematic nonholonomic model. The state $\boldsymbol{n}_i$ of agent i, $i=1,...,N$ consists of its position vector $\mathbf{n}_{i1}$ and orientation $\mathbf{n}_{i2}$:

$$\mathbf{n}_i = \begin{bmatrix} \mathbf{n}_{i1} \\ \mathbf{n}_{i2} \end{bmatrix}, \quad \mathbf{n}_{i1} = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \quad \mathbf{n}_{i2} = \begin{bmatrix} \phi_{i1} \\ \phi_{i2} \\ \phi_{i3} \end{bmatrix}$$

where $\begin{bmatrix} \phi_{i1} & \phi_{i2} & \phi_{i3} \end{bmatrix}^T$ are xyz Euler angles. This *Earth-fixed* coordinate system follow the *NED* (*North-East-Down*) convention with x_i pointing North, y_i East, and z_i Down. Consequently ϕ_{i1}, ϕ_{i2}, ϕ_{i3} express *bank*, *elevation* and *azimuth* angles of agent i respectively, as shown in Figure 1. We define the 3 body-fixed axes l_{i1}, l_{i2} and l_{i3}, pointing forward, right and downwards relatively to agent i respectively, as in Figure 2. Input $\mathbf{v}_i$ of each agent consists of the body-fixed, longitudinal velocity u_i (along axis l_{i1}) and the 3 earth-fixed angular velocities $\omega_{ik} = \dot{\phi}_{ik},\ k=1,2,3$:

$$\mathbf{v}_i = \begin{bmatrix} u_i & \omega_{i1} & \omega_{i2} & \omega_{i3} \end{bmatrix}^T$$

This selection of inputs resembles well the motion of an aircraft, as it does not allow any motion along the body-fixed lateral l_{i2} or perpendicular l_{i3} axes. The kinematics of the system described above are:

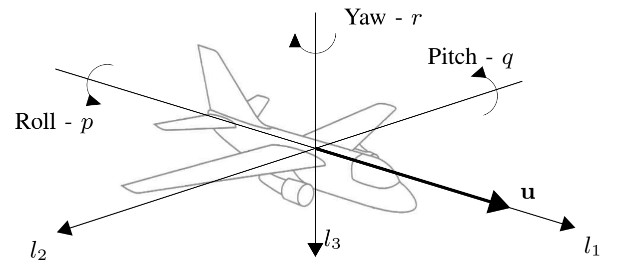

Fig. 2: Body-Fixed Coordinates

$$\dot{\mathbf{n}}_i = \mathbf{R}_i \cdot \mathbf{v}_i \tag{1}$$

with $\mathbf{R}_i = \mathbf{R}_i(\mathbf{n}_{i2}) \in \mathbb{R}^{6\times 4}$ the transformation matrix [14]:

$$\mathbf{R}_i = \begin{bmatrix} \mathbf{J}_i & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times1} & \boldsymbol{I}_3 \end{bmatrix}, \quad \mathbf{J}_i(\boldsymbol{n}_{i2}) = \begin{bmatrix} \cos\phi_{i3}\cos\phi_{i2} \\ \sin\phi_{i3}\cos\phi_{i2} \\ -\sin\phi_{i2} \end{bmatrix}$$

A. Problem Statement

The problem under consideration in this paper is to design a control law for each of N spherical agents. Each agent i has radius r_i and state $\boldsymbol{n}_i$, and is described by the kinematic model (1). The sought control law should steer each agent i having inputs: u_i, ω_{i1}, ω_{i2}, ω_{i3} to its desired position and direction (elevation and azimuth), $\boldsymbol{n}_{i1d}$ and ϕ_{i2d}, ϕ_{i3d} respectively, while avoiding collisions with each other or with the boundary ∂W of the given spherical workspace $W \subset \mathbb{R}^3$ of radius r_{world}. Each agent is assumed to have knowledge of the position, orientation and longitudinal velocity of all other agents, but not of their destinations. To comply with ATM requirements, we want to minimize the use of the agents' forward velocity for collision avoidance. Instead the forward velocity can be set to an independent, for each agent i, desired value u_{d_i}. This allows for example speed regulation based on Air Traffic Controller instructions, Flightplan or fuel efficiency for a given altitude.

The choice of a spherical workspace is considered here to allow direct application of the Navigation Functions framework, but does not limit the applicability of our algorithm. As shown in [15] navigation properties are maintained under diffeomorphisms. The reader is referred to [16] for details on the construction of such diffeomorphisms.

III. DIPOLAR NAVIGATION FUNCTIONS

Navigation Functions as introduced by Koditschek and Rimon [4] are not suitable for the control of non-holonomic agents, as they can lead to undesired behavior, like in-place rotation. *Dipolar Navigation Functions* [13] offer a significant advantage: the integral lines of the resulting potential field are all tangent to the target orientation at the destination, eliminating the need for in-place rotation. Thus, the agent is driven to its target with the desired orientation. This is achieved by considering the plane the normal vector of which is parallel to the desired orientation, and includes the destination, as an additional artificial obstacle H_{nh_i}.

Such a Dipolar Navigation Function has the form:

$$\Phi_i = \frac{\gamma_{di} + f_i}{\left((\gamma_{di} + f_i)^k + H_{nh_i} \cdot G_i \cdot \beta_{0_i}\right)^{1/k}} \tag{2}$$

where γ_{di} is the attractive destination function, G_i is the repulsive collision function, β_0 is the workspace boundary obstacle, f_i is used to allow some cooperation when needed between agents and k is a positive tuning parameter. Because of lack of space here, the reader is referred to [17] for details on the construction of the above Navigation Function.

Navigation Function (2) has been used in [5] and has proven properties, i.e., it provides almost global convergence to the agents' destinations, along with guaranteed collision avoidance. The potential of such a Navigation Function in a workspace with 2 obstacles is shown in Figure 3, where the effect of the nonholonomic obstacle H_{nh_i} is visible.

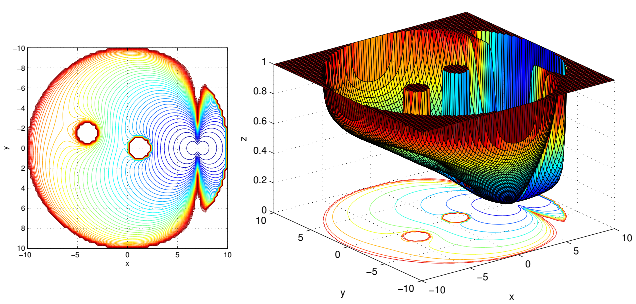

Fig. 3: 2-D Dipolar Navigation Function

IV. 3D Non-Holonomic Navigation

A. Control Law

The control scheme we suggest is based on the gradient $\nabla_i\Phi_i = \frac{\partial\Phi_i}{\partial \boldsymbol{n}_{i1}}$, where the notation $\nabla_i\Phi_j = \frac{\partial\Phi_j}{\partial \boldsymbol{n}_{i1}}$ stands for the gradient of potential Φ_j with respect to agent's i position $\boldsymbol{n}_{i1}$. The potential $\Phi_i = \Phi_i(n_{i1})$ is the above *Dipolar Navigation Function* (2). Since its gradient $\nabla_i\Phi_i$ is expressed in earth-fixed coordinates, we use the projection of $\nabla_i\Phi_i$ on agent's i longitudinal axis l_{i1}: $P_i = \boldsymbol{J}_i^T \cdot \nabla_i\Phi_i$, the sign $\mathrm{sgn}(P_i)$ of which determines the direction of movement, using the modified sign function sgn:

$$\mathrm{sgn}(x) \triangleq \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$$

For the angular velocities control laws we use the nonholonomic elevation and azimuth angles $\phi_{\mathbf{nh}i2}$, $\phi_{\mathbf{nh}i3}$ representing the direction of $\mathrm{sgn}(p_i)\nabla_i\Phi_i$:

$$\phi_{\mathbf{nh}i3} \triangleq atan2\left(\mathrm{sgn}(p_i)\Phi_{iy}, \mathrm{sgn}(p_i)\Phi_{ix}\right) \tag{3a}$$

$$\phi_{\mathbf{nh}i2} \triangleq atan2\left(\text{-}\mathrm{sgn}(p_i)\Phi_{iz}, \sqrt{\Phi_{ix}^2 + \Phi_{iy}^2}\right) \tag{3b}$$

where $\mathrm{atan2}(y,x) \triangleq arg\,(x,y)\,, \quad (x,y) \in \mathbb{C}$ yields the polar angle of a vector. $\Phi_{ix} = \frac{\partial\Phi_i}{\partial x_i}$, $\Phi_{iy} = \frac{\partial\Phi_i}{\partial y_i}$, $\Phi_{iz} = \frac{\partial\Phi_i}{\partial z_i}$ are the partial derivatives of Φ_i with respect to agent's i position n_{i1} and $p_i = \mathbf{J}_{id}^T \cdot (\boldsymbol{n}_{i1} - \boldsymbol{n}_{i1d})$ is the current position vector with respect to the destination, projected on the longitudinal axis of the desired orientation (l_{i1d}). We also define the nonholonomic bank angle $\phi_{\mathbf{nh}i1}$, designed to minimize yaw rotation rate, as explained in [18]:

$$\phi_{\mathbf{nh}i1} \triangleq atan2\left(sgn(p_i)\, c\phi_{i2} \cdot \omega_{i3}, sgn(p_i)\, \omega_{i2}\right)$$

The discontinuity of the above angles at the destination, where the gradient vector is zero, can be avoided by using the approximation scheme presented in [19]:

$$\hat{\phi}_{\mathbf{nh}ik} \triangleq \begin{cases} \phi_{\mathbf{nh}ik}, & \rho_{ik} > \epsilon \\ \frac{\phi_{\mathbf{nh}ik}\left(-2\rho_{ik}^3 + 3\epsilon\rho_{ik}^2\right) + \phi_{ikd}\left(-2(\epsilon-\rho_{ik})^3 + 3\epsilon(\epsilon-\rho_{ik})^2\right)}{\epsilon^3}, & \rho_{ik} \leq \epsilon \end{cases}$$

for $k = 1,2,3$, where $\rho_{i1} = \sqrt{c\phi_{i2}^2 \cdot \omega_{i3}^2 + \omega_{i2}^2}$, $\rho_{i2} = ||\nabla_i\Phi_i||$, $\rho_{i3} = \sqrt{\Phi_{ix}^2 + \Phi_{iy}^2}$ and ϵ a small positive constant. Thus at the destination $\boldsymbol{n}_{i1d}$, where $\rho_{ik} = 0$, $k = 1,2,3$, the angles $\hat{\phi}_{\mathbf{nh}ik}$ are continuous and equal to the desired ones:

$$\hat{\phi}_{\mathbf{nh}ik} = \phi_{ikd}, \ \ k = 1,2,3 \tag{4}$$

We also use the partial derivative $\dfrac{\partial\Phi_i}{\partial t} = \sum\limits_{j \neq i} u_j \nabla_j \Phi_i^T \cdot \boldsymbol{J}_j$ which sums the effect of all other agents' motion on Φ_i.

The proposed control law for agent i, $i = 1,\ldots,N$ is:

$$u_i = \begin{cases} -\,\mathrm{sgn}(P_i)\cdot U_i, & \frac{\partial\Phi_i}{\partial t} \leq U_i\left(|P_i| - \varepsilon\right) \\ -\frac{1}{P_i}\left[\varepsilon U_i + \frac{\partial\Phi_i}{\partial t}\right], & \frac{\partial\Phi_i}{\partial t} > U_i\left(|P_i| - \varepsilon\right) \end{cases} \tag{5a}$$

$$U_i = \begin{cases} u_{d_i}, & ||\boldsymbol{n}_{i1} - \boldsymbol{n}_{i1d}|| > \rho \\ \frac{||\boldsymbol{n}_{i1} - \boldsymbol{n}_{i1d}||}{\rho} \cdot u_{d_i}, & ||\boldsymbol{n}_{i1} - \boldsymbol{n}_{i1d}|| \leq \rho \end{cases} \tag{5b}$$

$$\omega_{i1} = -\,k_{i1}\left(\phi_{i1} - \phi_{\mathbf{nh}i1}\right) \tag{5c}$$

$$\omega_{ik} = \begin{cases} 0, & M_{ik} \geq \varepsilon_k \\ \Omega_{ik}\cdot\left(1 - \frac{M_{ik}}{\varepsilon_k}\right), & 0 < M_{ik} < \varepsilon_k\ ,\ k = 2,3 \\ \Omega_{ik}, & M_{ik} \leq 0 \end{cases} \tag{5d}$$

where U_i is the nominal absolute velocity value, $\Omega_{ik} \triangleq -k_k\left(\phi_{ik} - \phi_{\mathbf{nh}ik}\right) + \dot{\phi}_{\mathbf{nh}ik}$, $M_{ik} \triangleq \dot{\phi}_{\mathbf{nh}ik}\left(\phi_{ik} - \phi_{\mathbf{nh}ik}\right)$ while ε, ε_k for $k = 2,3$ are small positive constants and k_k are positive real gains.

Control design principle

The notion behind the above control strategy is to exploit the Navigation Function's almost global navigation and collision avoidance properties, while keeping the velocity input as independent as possible from the potential field's qualitative characteristics. Thus the direction of the gradient is used to steer the agent and define its direction of movement, but the gradient's norm is used for the regulation of velocity only when necessary. It is easy to verify that each agent i moves towards the direction of its longitudinal axis that decreases its Navigation Function Φ_i: by (5a) we can see that $u_i \cdot P_i \leq 0$, i.e. the velocity u_i is always in the opposite direction of P_i (the gradient's $\nabla_i\Phi_i$ projection on the agent's longitudinal axis). Thus collision avoidance is guaranteed, as the potential field is always repulsive with respect to other agents and achieves its maximum value on their boundary. For the regulation of the linear velocity a desired value u_{d_i} is used, which is applied whenever the agent is outside of a ball of radius ρ around its destination $\boldsymbol{n}_{i1d}$, and $\frac{\partial \Phi_i}{\partial t} \leq U_i\left(|P_i| - \varepsilon\right)$ holds. This last condition essentially ensures that the partial derivative $\frac{\partial \Phi_i}{\partial t}$ is not larger than what the motion of the agent can negate, allowing its Navigation Function decreases over time. The desired velocity u_{d_i} can be either constant, or regulated independently, and acts as a lower bound of the agent's absolute velocity $|u_i|$, as when $\frac{\partial \Phi_i}{\partial t} > U_i\left(|P_i| - \varepsilon\right)$ we deduce that $|u_i| = \left|\frac{1}{P_i}\left[\varepsilon U_i + \frac{\partial \Phi_i}{\partial t}\right]\right| > U_i$. This is extremely important in applications where the agents' velocity cannot be lower than a certain value, like aircraft navigation. We should note that the switching in (5a) and (5b) is continuous: u_i and U_i are continuous when $\frac{\partial \Phi_i}{\partial t} = U_i\left(|P_i| - \varepsilon\right)$ and $||\boldsymbol{n}_{i1} - \boldsymbol{n}_{i1d}|| = \rho$ respectively.

Regarding the control law for elevation and azimuth angular velocities (5d), we have introduced a switching scheme in order to limit unnecessary maneuvering: Whenever $M_{ik} \geq \varepsilon_k > 0$, the absolute angle difference $|\theta_{ik}|$, $\theta_{ik} = (\phi_{ik} - \phi_{\mathbf{nh}ik})$ is decreasing without the need of any actuation ω_{ik} . When $M_{ik} \leq 0$, the control law used in previous work of the authors [18] is employed. Similarly to the linear velocity control scheme, we use a small positive constant ε_k to ensure continuous transition from zero angular velocity to the full actuation Ω_{ik}. The magnitude of ε_k adjusts the tolerance of the algorithm to small, but decreasing rates of change for $|\theta_{ik}|$. Setting ε_k to a very small value allows ω_{ik} to remain zero even for very small, but positive M_{ik}. Similarly, a very large ε_k essentially prevents the use of zero ω_{ik}, as $M_{ik} \geq \varepsilon_k$ cannot hold. This switching control scheme is not intended to provide an optimal selection of angular velocities, but as the simulation results of the next section reveal, it does manage to limit the total control effort for steering. Calculating the dynamics of θ_{ik} we deduce:

$$\dot{\theta}_{ik} = \begin{cases} -\dot{\phi}_{\mathbf{nh}ik}, & M_{ik} \geq \varepsilon_k \\ -\left[k_k\left(1 - \frac{M_{ik}}{\varepsilon_k}\right) + \frac{\dot{\phi}^2_{\mathbf{nh}ik}}{\varepsilon_k}\right] \cdot \theta_{ik}, & 0 < M_{ik} < \varepsilon_k \\ -k_k \theta_{ik}, & M_{ik} \leq 0 \end{cases}$$

Therefore the absolute difference $|\theta_{ik}|$ is always decreasing and the agents align with the field's gradient. Finally, the bank rotation rate used is designed to keep the need for yaw rotation rate low, as explained in [18].

Avoidance of infinite velocity - direction reversal

From the control law (5a) we can see that the velocity u_i can tend to infinite values when $P_i \to 0$, i.e., when the projection of the field's gradient on the agent's longitudinal axis is very small. This is the case when the gradient vector is normal to the agent's longitudinal axis: $\nabla_i \Phi_i \perp l_{i1}$. By the analysis in the previous paragraph, θ_{ik} is stabilised to 0, and $|\theta_{ik}|$ is always decreasing for $k = 2, 3$. Therefore if the absolute angle between the field's gradient and l_{i1} is initially smaller than $\frac{\pi}{2}$, it will always remain in $\left[0, \frac{\pi}{2}\right)$. Thus the set $\nabla_i \Phi_i \perp l_{i1}$, where $P_i \to 0$, will never be reached. Essentially, what is required is $P_i \cdot p_i > 0$ at the initial conditions, i.e. agents starting in the subspace behind their targets (where $p_i < 0$) must have the initial negated gradient vector driving them forward ($P_i < 0$), while agents starting in front of their target ($p_i > 0$) must have the initial negated gradient vector driving them backward ($P_i > 0$). If we want additionally to enforce only forward (or backward) motion, we have to ensure that all agents start in the subspace behind (in front) of their target. These mild prerequisites should not pose considerable difficulties in air traffic applications, since they represent natural requirements.

B. Stability Analysis

Theorem 1: Each agent i described by model (1) under the control law (5) is asymptotically stabilised to its target $\boldsymbol{n}_{i1d}$, ϕ_{i2d}, ϕ_{i3d}.

Proof: As the control scheme is discontinuous, we will use Lyapunov analysis for nonsmooth systems to prove the stability of the system under the control law (5). The following Lyapunov function candidate is used:

$$V = \sum_{i=1}^{N} V_i, \quad V_i = \Phi_i + \frac{1}{2}\sum_{k=2}^{3}\left(\phi_{ik} - \phi_{\mathbf{nh}ik}\right)^2 \qquad (6)$$

The generalised derivative of $V = V(\mathbf{q})$ [20], where $\mathbf{q} = \left[\boldsymbol{n}_{i1}^T \dots \boldsymbol{n}_{N1}^T \; \phi_{12} \; \phi_{13} \dots \phi_{N2} \; \phi_{N3} \; \phi_{\mathbf{nh}12} \; \phi_{\mathbf{nh}13} \dots \phi_{\mathbf{nh}N2} \; \phi_{\mathbf{nh}N3}\right]^T$

$$\text{is } \partial V = \begin{bmatrix} \sum_i \nabla_1 \Phi_i \\ \vdots \\ \sum_i \nabla_N \Phi_i \\ {}^1/_2 \nabla_{\phi_{12}} (\phi_{12} - \phi_{\mathbf{nh}12})^2 \\ {}^1/_2 \nabla_{\phi_{13}} (\phi_{13} - \phi_{\mathbf{nh}13})^2 \\ \vdots \\ {}^1/_2 \nabla_{\phi_{N2}} (\phi_{N2} - \phi_{\mathbf{nh}N2})^2 \\ {}^1/_2 \nabla_{\phi_{N3}} (\phi_{N3} - \phi_{\mathbf{nh}N3})^2 \\ {}^1/_2 \nabla_{\phi_{\mathbf{nh}12}} (\phi_{12} - \phi_{\mathbf{nh}12})^2 \\ {}^1/_2 \nabla_{\phi_{\mathbf{nh}13}} (\phi_{13} - \phi_{\mathbf{nh}13})^2 \\ \vdots \\ {}^1/_2 \nabla_{\phi_{\mathbf{nh}N2}} (\phi_{N2} - \phi_{\mathbf{nh}N2})^2 \\ {}^1/_2 \nabla_{\phi_{\mathbf{nh}N3}} (\phi_{N3} - \phi_{\mathbf{nh}N3})^2 \end{bmatrix} = \begin{bmatrix} \sum_i \nabla_1 \Phi_i \\ \vdots \\ \sum_i \nabla_N \Phi_i \\ (\phi_{12} - \phi_{\mathbf{nh}12}) \\ (\phi_{13} - \phi_{\mathbf{nh}13}) \\ \vdots \\ (\phi_{N2} - \phi_{\mathbf{nh}N2}) \\ (\phi_{N3} - \phi_{\mathbf{nh}N3}) \\ -(\phi_{12} - \phi_{\mathbf{nh}12}) \\ -(\phi_{13} - \phi_{\mathbf{nh}13}) \\ \vdots \\ -(\phi_{N2} - \phi_{\mathbf{nh}N2}) \\ -(\phi_{N3} - \phi_{\mathbf{nh}N3}) \end{bmatrix}$$

We consider the multi-agent system $\dot{\mathbf{x}} = f(\mathbf{x})$ resulting from the composition of (1) and the associated Filippov set [21]:

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{n}_{11} \\ \vdots \\ \boldsymbol{n}_{N1} \\ \phi_{12} \\ \phi_{13} \\ \vdots \\ \phi_{N2} \\ \phi_{N3} \\ \phi_{\mathbf{nh}12} \\ \phi_{\mathbf{nh}13} \\ \vdots \\ \phi_{\mathbf{nh}N2} \\ \phi_{\mathbf{nh}N3} \end{bmatrix}, \quad f(\mathbf{x}) = \begin{bmatrix} u_1 \boldsymbol{J}_1 \\ \vdots \\ u_N \boldsymbol{J}_N \\ \omega_{12} \\ \omega_{13} \\ \vdots \\ \omega_{N2} \\ \omega_{N3} \\ \dot{\phi}_{\mathbf{nh}12} \\ \dot{\phi}_{\mathbf{nh}13} \\ \vdots \\ \dot{\phi}_{\mathbf{nh}N2} \\ \dot{\phi}_{\mathbf{nh}N3} \end{bmatrix}, \quad K[f] = \begin{bmatrix} K[u_1] \boldsymbol{J}_1 \\ \vdots \\ K[u_N] \boldsymbol{J}_N \\ \omega_{12} \\ \omega_{13} \\ \vdots \\ \omega_{N2} \\ \omega_{N3} \\ \dot{\phi}_{\mathbf{nh}12} \\ \dot{\phi}_{\mathbf{nh}13} \\ \vdots \\ \dot{\phi}_{\mathbf{nh}N2} \\ \dot{\phi}_{\mathbf{nh}N3} \end{bmatrix}$$

By the control law (5a) we deduce:

$$K[u_i] = \begin{cases} K[-sgn(P_i)] \cdot U_i, & \frac{\partial \Phi_i}{\partial t} \leq U_i\left(|P_i| - \varepsilon\right) \\ -\frac{1}{P_i} \cdot \left[\varepsilon U_i + \frac{\partial \Phi_i}{\partial t}\right], & \frac{\partial \Phi_i}{\partial t} > U_i\left(|P_i| - \varepsilon\right) \end{cases} \qquad (7)$$

Using the chain rule given in [22] we can calculate the generalised time derivative of V as follows:

$$\begin{aligned}\dot{\tilde{V}} &= \bigcap_{\xi\in\partial V}\xi^T K[f] = \\ &=\sum_i\sum_j K[u_i]\nabla_i\Phi_j^T \boldsymbol{J}_i + \\ &\quad+\sum_i\sum_{k=2}^{3}\left(\phi_{ik}-\phi_{\boldsymbol{nh}_{ik}}\right)\left(\omega_{ik}-\dot{\phi}_{\boldsymbol{nh}_{ik}}\right) = \\ &=\sum_i K[u_i]\nabla_i\Phi_i^T\boldsymbol{J}_i + \sum_i\sum_{j\neq i}K[u_j]\nabla_j\Phi_i^T\boldsymbol{J}_j - \\ &\quad-\sum_i\sum_{k=2}^{3}\theta_{ik}\dot{\theta}_{ik}\end{aligned}$$

Because of the switching linear velocity control law, we discriminate between the following sets of agents:
$Q_1 \triangleq \left\{i\in\{1,\ldots,N\}\,\middle|\,\frac{\partial\Phi_i}{\partial t}\le U_i\left(|P_i|-\varepsilon\right)\right\}$
$Q_2 \triangleq \left\{i\in\{1,\ldots,N\}\,\middle|\,\frac{\partial\Phi_i}{\partial t}> U_i\left(|P_i|-\varepsilon\right)\right\}$
with $Q_1\bigcap Q_2=\varnothing$. Similarly, we define the following non-intersecting sets of (i,k) pairs:
$T_1 \triangleq \{i\in\{1,\ldots,N\},k\in\{2,3\}\mid M_{ik}\ge\varepsilon_k\}$
$T_2 \triangleq \{i\in\{1,\ldots,N\},k\in\{2,3\}\mid 0<M_{ik}<\varepsilon_k\}$
$T_3 \triangleq \{i\in\{1,\ldots,N\},k\in\{2,3\}\mid M_{ik}\le 0\}$

Using the above set definitions, and the calculation of $\dot{\theta}_{ik}$ in the previous chapter, we can proceed with $\dot{\tilde{V}}$:

$$\begin{aligned}\dot{\tilde{V}} &\overset{(7)}{=} \sum_{Q_1}\left\{K[-sgn(P_i)]\cdot P_iU_i+\frac{\partial\Phi_i}{\partial t}\right\} - \\ &\quad-\sum_{Q_2}\left\{\frac{1}{P_i}\left[\varepsilon U_i-\frac{\partial\Phi_i}{\partial t}\right]P_i+\frac{\partial\Phi_i}{\partial t}\right\}-\sum_{T_1}\dot{\phi}_{\mathbf{nh}ik}\theta_{ik} - \\ &\quad-\sum_{T_2}\left[k_k\left(1-\frac{M_{ik}}{\varepsilon_k}\right)+\frac{\dot{\phi}^2_{\mathbf{nh}ik}}{\varepsilon_k}\right]\theta_{ik}^2-\sum_{T_3}k_k\theta_{ik}^2 = \\ &=\sum_{Q_1}\left\{-\left|P_i\right|U_i+\frac{\partial\Phi_i}{\partial t}\right\}-\sum_{Q_2}\varepsilon U_i-\sum_{T_1}M_{ik} - \\ &\quad-\sum_{T_2}\left[k_k\left(1-\frac{M_{ik}}{\varepsilon_k}\right)\theta_{ik}^2+\frac{M_{ik}^2}{\varepsilon_k}\right]-\sum_{T_3}k_k\theta_{ik}^2\end{aligned}$$

Taking into account the conditions that hold within each set, we derive that $\dot{\tilde{V}}\le 0$. Since each V_i and consequently V is regular [20] and the level sets of V are compact, the nonsmooth version of LaSalle's invariance principle [22] can be applied. We can conclude that the trajectory of the closed-loop system converges to the largest invariant subset S: $S\triangleq\left\{\mathbf{n}\,|\,0\in\dot{\tilde{V}}\right\}$. By the definitions of T_1, T_2, T_3 we deduce that $\sum_{T_1}M_{ik}>0$ and $\sum_{T_2}\left[k_k\left(1-\frac{M_{ik}}{\varepsilon_k}\right)\theta_{ik}^2+\frac{M_{ik}^2}{\varepsilon_k}\right]>0$. Consequently, for $\dot{\tilde{V}}=0$ to hold, all (i,k) must be in T_3. Therefore the set S is:

$$\begin{aligned}S=&\{\mathbf{n}:(\left|P_i\right|U_i-\frac{\partial\Phi_i}{\partial t}=0\forall i\in Q_1)\wedge(\varepsilon U_i=0\forall i\in Q_2)\wedge \\ &\wedge(\theta_{ik}=\phi_{ik}-\phi_{\boldsymbol{nh}_{ik}}=0\forall i,k=2,3)\}\end{aligned}$$

For $i\in Q_1$ we have $\left|P_i\right|U_i-\frac{\partial\Phi_i}{\partial t}\ge\varepsilon U_i$, so inside S the equality must hold, as εU_i is always non-negative and zero iff $U_i=0$. Similarly, for $\varepsilon U_i=0$ to hold for i/inQ_2, U_i must be zero too. Thus, inside S we have $U_i=0$, and $\phi_{ik}=\phi_{\boldsymbol{nh}_{ik}}\forall i,k=2,3$. Condition $U_i=0$ holds only when $\boldsymbol{n}_{i1}=\boldsymbol{n}_{i1d}$, i.e. when each agent i has reached its target position $\boldsymbol{n}_{i1}$. Finally, by (4) and condition $\phi_{ik}=\phi_{\boldsymbol{nh}_{ik}}\forall i,k=2,3$, we deduce that the set S reduces to the singleton $\{\mathbf{n}:(\boldsymbol{n}_{i1}=\boldsymbol{n}_{i1d}\forall i)\wedge(\phi_{ik}=\phi_{ikd}\forall i,k=2,3)\}$, i.e., all agents are stabilised to their destinations with the desired elevation and azimuth angles. ■

V. Simulation

The proposed control scheme has been used in a simulated scenario in order to verify its performance. We used a test case with 5 agents of radius $r_i=0.05$, $i=1,\ldots,5$. The initial positions and destinations have been selected so that the straight line paths create multiple conflicts around the centre of the workspace. The desired velocity of all the agents has been set to a constant value of $5\cdot10^{-4}$, while for the final approach we have used $\rho=0.01$.

The results can be seen in Figure V, with all agents being driven towards their destinations following feasible, 3D nonholonomic paths. The efficiency of the control scheme presented here can be seen in Figure 5, where the linear velocity of all agents is maintained equal to the constant desired value, until they approach their targets and start to slow down as intended. All the agents started with their destinations in front of them, and the initial velocity vector pointed forward too. As explained in Section IV, this resulted in all the agents moving forward without any direction reversals. In order to demonstrate the efficiency of the angular velocities control law, we run the same simulation after setting ε to a very large value. As discussed in Section IV, this effectively prevents the use of zero angular velocities at any time. To asses the performance in those two cases we used the absolute total control effort used in elevation and azimuth by all agents during the simulation: $A=\sum_{i=1}^{5}\sum_{k=2}^{3}\int\left|\omega_{ik}\right|dt$. The control scheme used in previous work [6] resulted in $A=11.4rad$, while the one introduced here yielded $A=18.8rad$, achieving a significant reduction of about 40% in steering control effort. It is important to note that these qualitative improvements did not affect the collision avoidance characteristics of the control strategy. As shown in Figure 6, distances between agents (solid lines) are always higher than the minimum safety clearance (dashed line), which is double the radius of each agent $2\cdot r_i=0.1$, and no collisions occur.

VI. Conclusions

We have presented a decentralised control strategy that drives nonholonomic, aircraft-like agents towards their desired configuration with a desired linear velocity varied independently. This desired linear velocity, constant or otherwise regulated, is directly applied most of the time and defines a lower bound for the absolute actual velocity. The angular velocities used can be zero when not required, thus reducing the total required control effort. It is interesting to note here

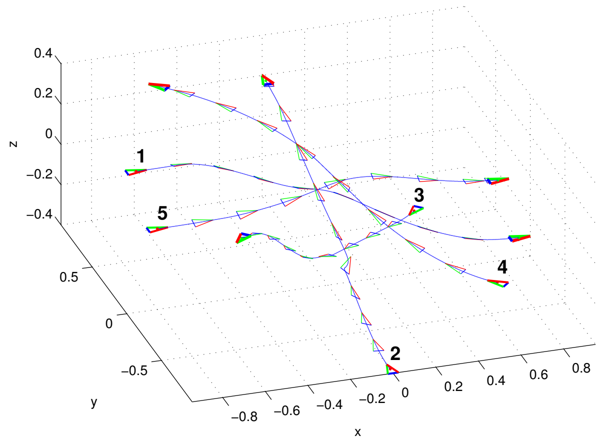

Fig. 4: Agents' Trajectories in 3D space

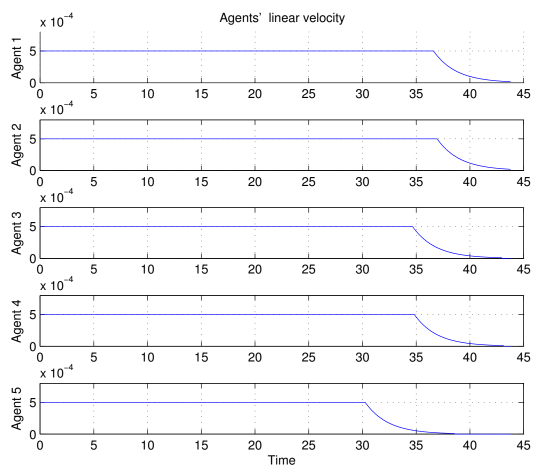

Fig. 5: Control inputs applied to the agents

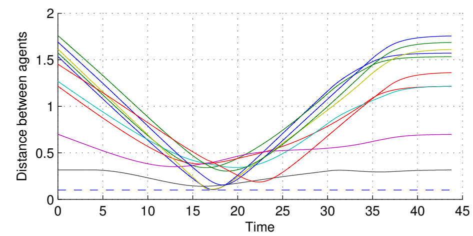

Fig. 6: Distances between agents (solid lines) and minimum safety clearance (dashed line)

that if the agents are not able to alter their linear velocity from the constant desired value u_{d_i}, the collision avoidance guarantees still hold, as $u_i \cdot P_i \leq 0$ and the Navigation Function is always repulsive with respect to other agents. In this case though, the stability analysis presented here is not valid and therefore convergence is not guaranteed.

Future work in this area focuses on incorporating additional constraints, like a maximum curvature bound, to enhance the applicability of our algorithm in aircraft control.

VII. Acknowledgements

The authors of this paper want to acknowledge the contribution of the European Commission through project iFLY.

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